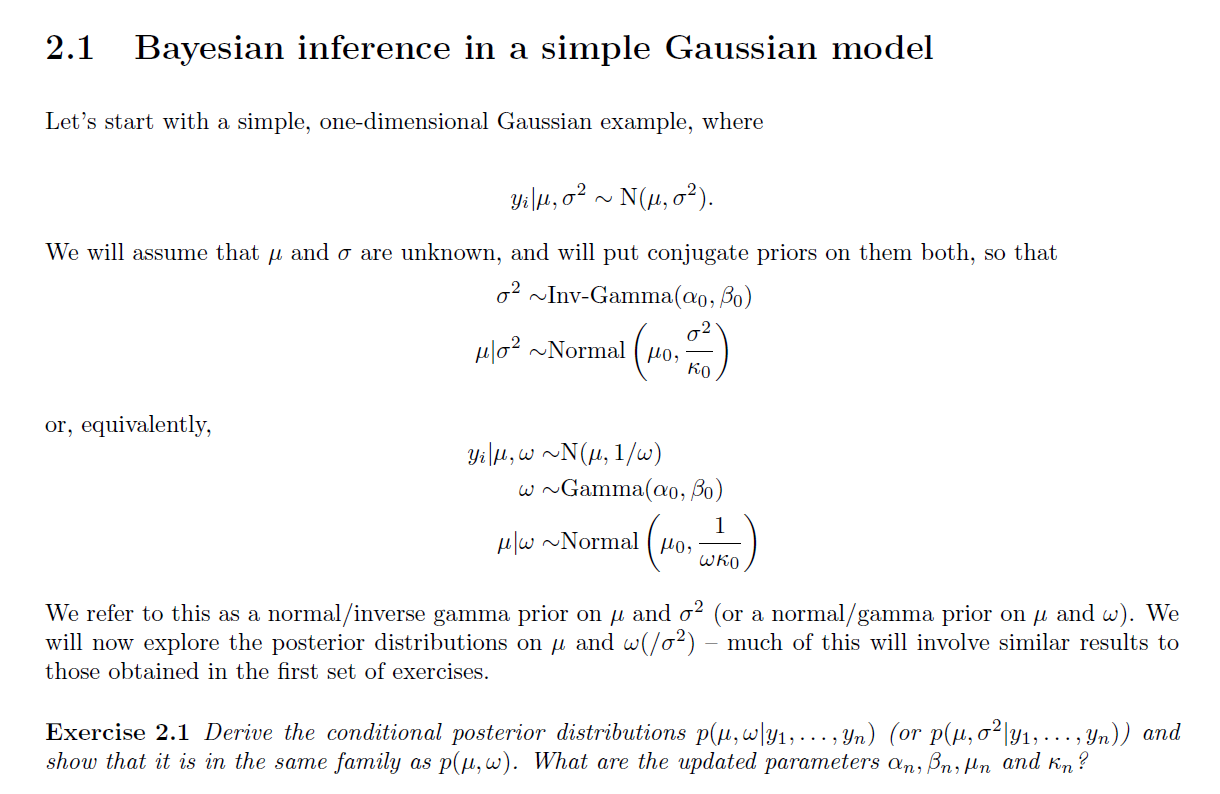
**Sareh Kouchaki – Section 2 Statistical-Modeling-II**



Solution

First, we need to find the likelihood:

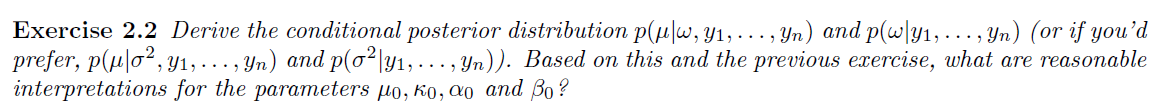
From the question we have the prior on and the prior on , Conjugate prior on is as following:

Posterior distribution is proportional to the product of likelihood and conjugate prior:

\*\*

So,

Normal Gamma distribution,



Solution

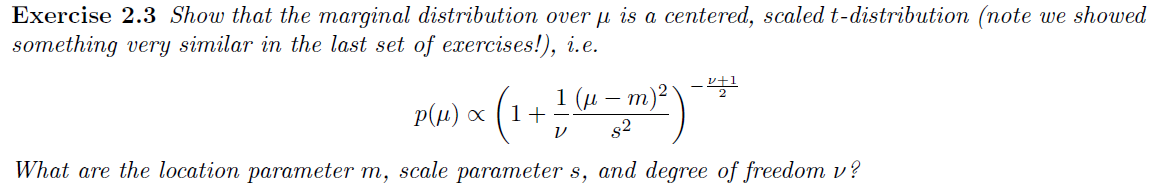
First, we need to find which is equal to

Next, we need to find which is equal to:

So,

The above function is the kernel of normal distribution. Therefore, conditional posterior distribution of is:

Reasonable interpretation on parameters



Solution

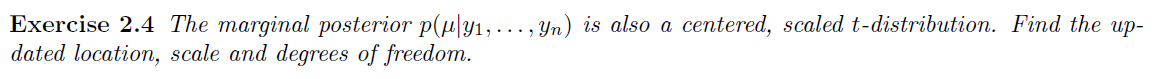
Marginal distribution over is obtained through following:

Under the integral is the kernel of gamma ( distribution

where .

So,

a scaled t distribution where



Solution

Under the integral is the kernel of gamma ( distribution

where .

So,

Scaled t distribution with



Solution

It is better to first solve the exercise 2.6 and then solve this exercise.

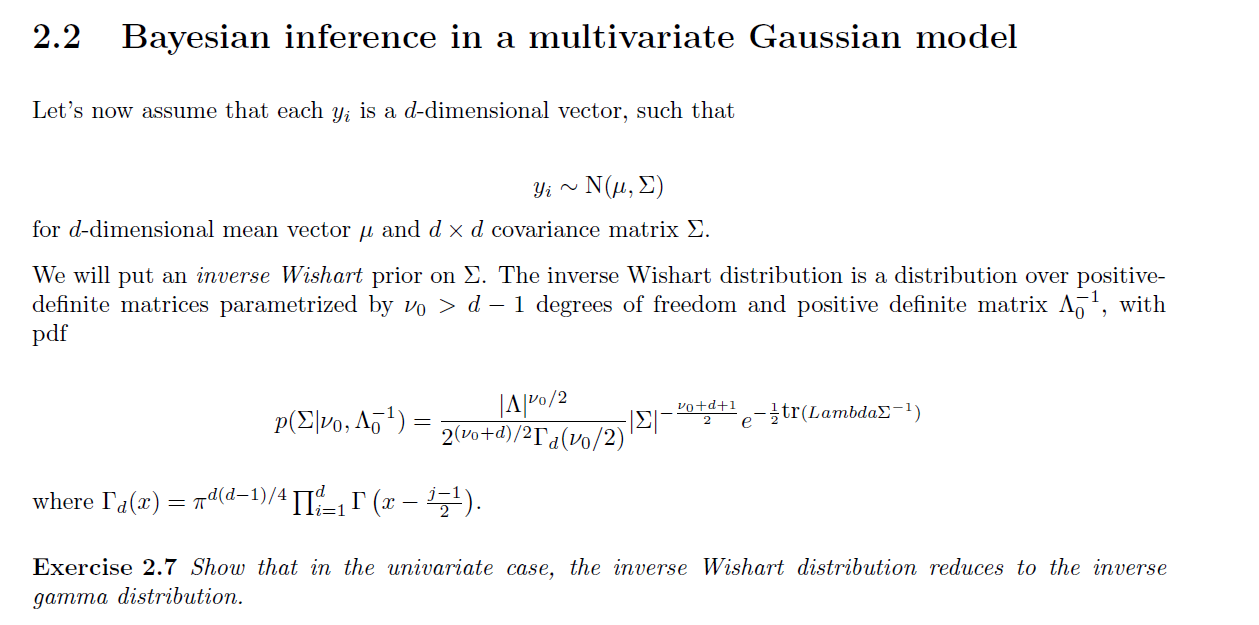
from the exercise 2.6 we found the marginal distribution over Y. So, we can write



Solution:

where

If we divide the above equation by the posterior distribution:



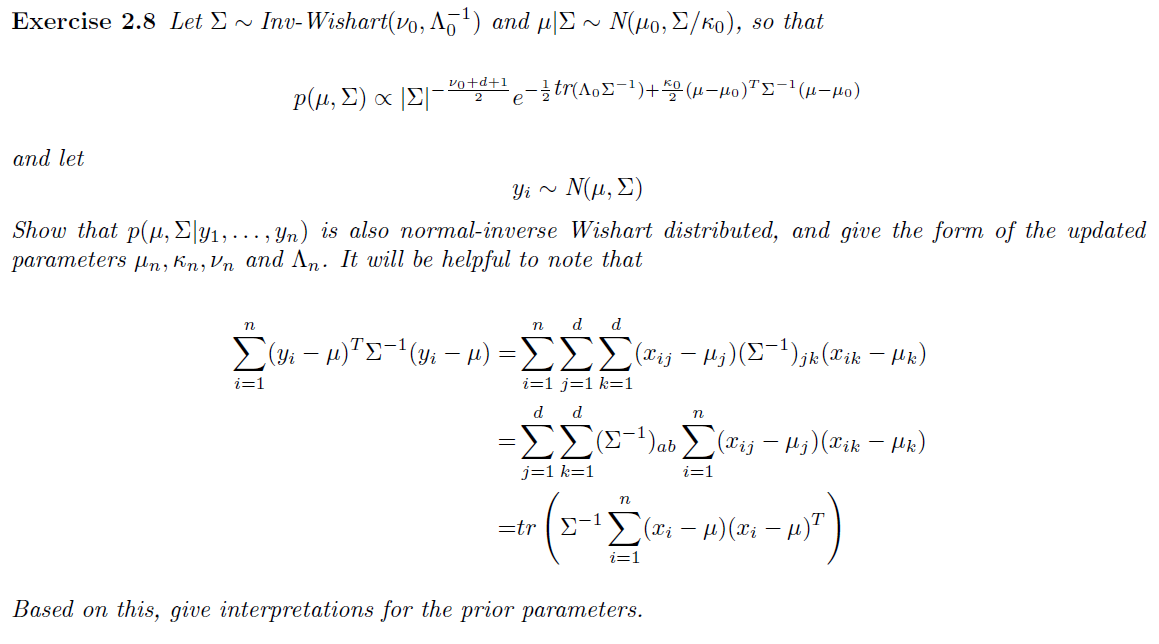
Solution:

Probability density function of the inverse Wishart is as follows:

Where and are d\*d positive definite matrices, and is the multivariate gamma function.

With d=1, , the probability density function of inverse Wishart becomes:

= Inverse Gamma (



Solution

Posterior distribution can be written as following:

From the question we have:

So,

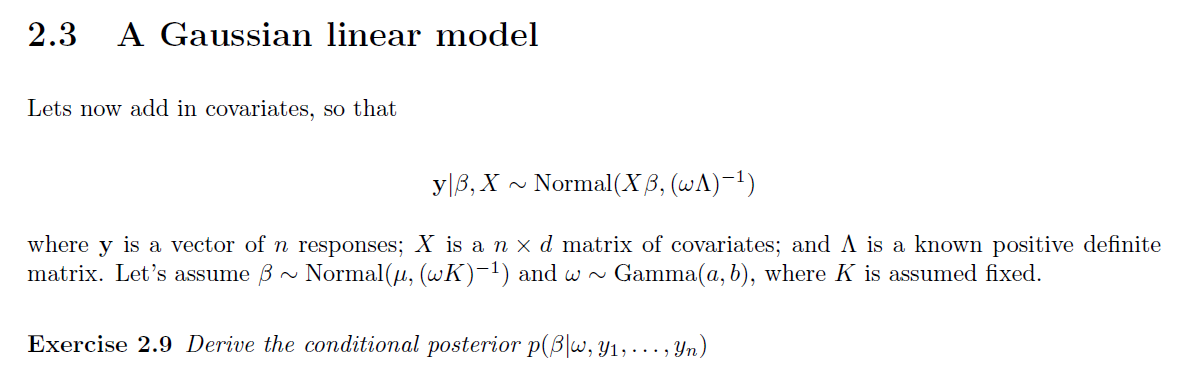
=

\*\*

\*\*

So, the posterior is normal inverse Wishart with the following updated parameters:

Interpretations for the prior parameter:



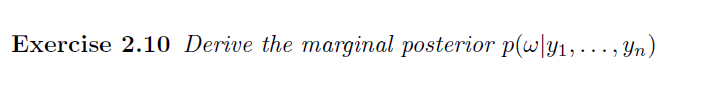
Solution

From the question we have:

We want to find

So, we can write

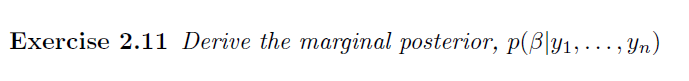
So, this distribution is normal with mean of and variance of .



Solution:

\*\*

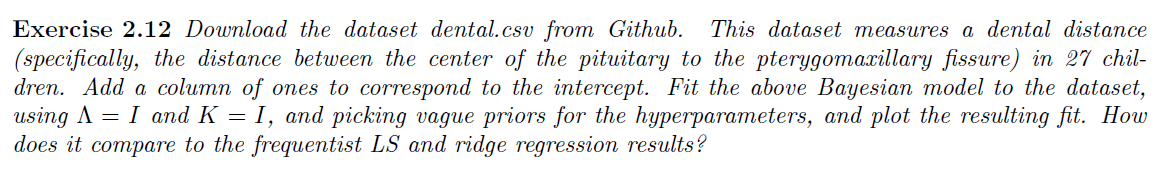
A gamma distribution with the updated parameters of



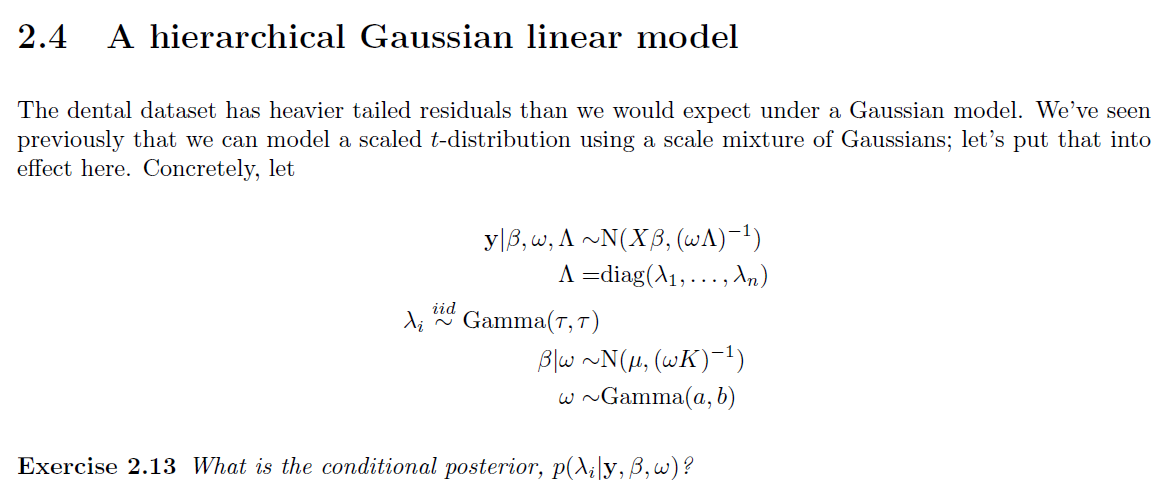
Solution:

Kernel of Gamma Distribution

t distribution with 2 degrees of freedom.



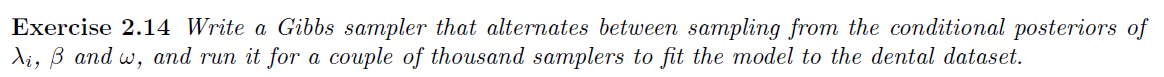
R Code is on GitHub.



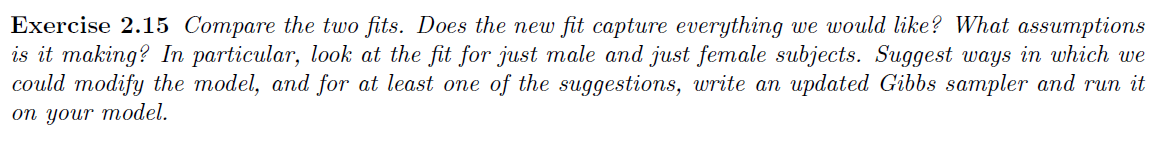
Solution:

\* is independent of .

So, Gamma( , ).



R code is on the GitHub.



Solution

No, comparing to the linear regression, we can say that the new fit does not capture everything we would like to capture.

How to modify the model?